Math 75 Test 3 take-home quiz

SOLUTION KEY

1. What is the minimum value of the function $f(t) = 3t^3 - 3t^2 - 12t + 6$ on the interval $[-2,3]$ ?

Checking endpoint values first: $f(3) = 24$

$f(-2) = -6$

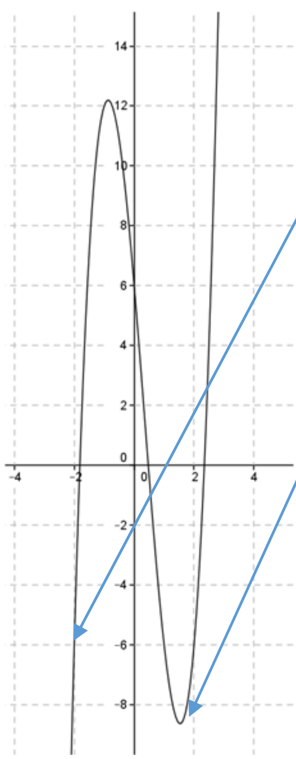
Now taking derivative:

$f'(x) = 9t^2 - 6t - 12 = 0 \rightarrow 3t^2 - 2t - 4 = 0 \rightarrow t = \frac{1 \pm \sqrt{13}}{3}$ which gives two solutions in the interval, $t \approx 1.535$ and $t \approx -0.868$.

$f(-0.868) \approx$ 12.94 and $f(1.535) \approx$ -8. 6383 < -6

Hence the minimum value is approximately -8. 6383. On an exam, the numbers would be easier, because you do not have a calculator.

Here is a graph just see understand more what just happened:



2. A company wants to make a box with a lid that has a square base and a volume of 17,576 cubic inches. Find the dimensions and the surface area of the box that uses the least amount of material.

$V = x^2 y = 17576$ and $SA = x^2 + x^2 + 4xy = 2x^2 + 4x\frac{17576}{x^2} = 2x^2 + \frac{70304}{x}$

Now set the derivative to zero:

$$(SA)' = 4x - \frac{70304}{x^2} = 0 \to x^3 = 17576 \to x = 26$$

For dimensions of box, solve for $y$: $y = \frac{17576}{26^2} = 26$ so it is a $26 \times 26 \times 26$ box and lid;

Having surface area of : $SA_{min} = 2(26)^2 + 4(26)(26) = 6 \cdot 26^2 = 4056\ in^2$

Note: if the problem said 'without lid' then the volume constraint equation would not change, but the surface area equation would now be: $SA = x^2 + \frac{70304}{x}$ since there is no lid!

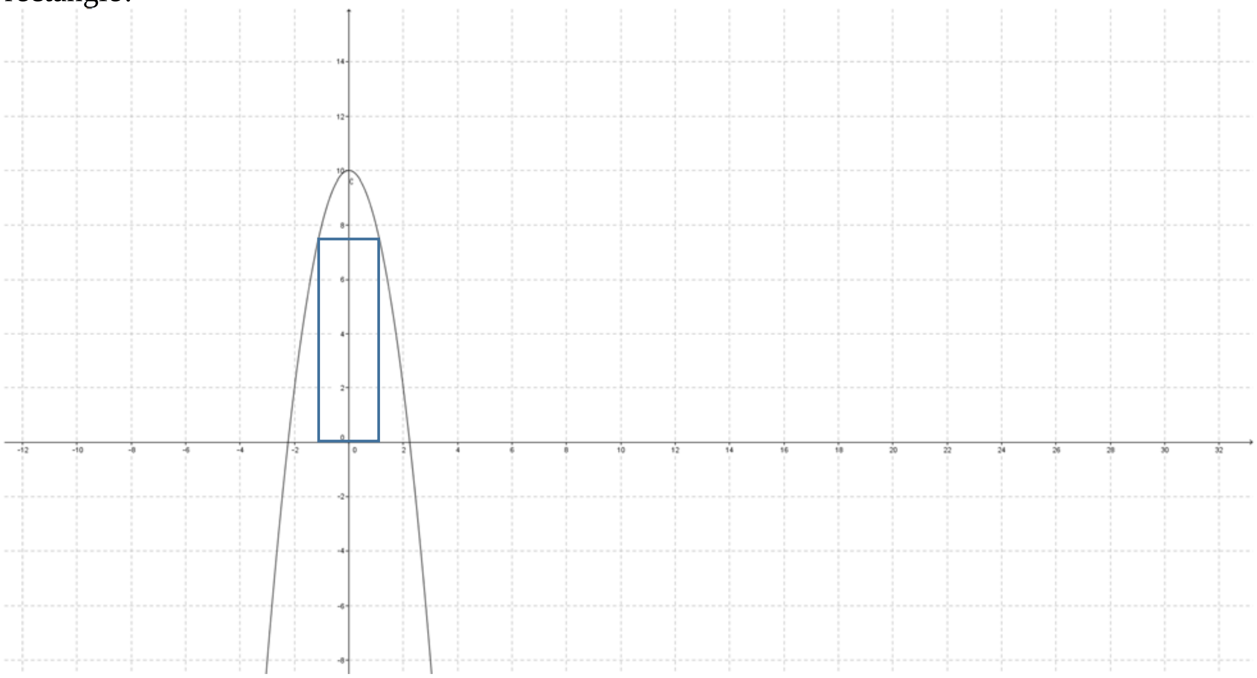
3. Use linear approximation to estimate $\sqrt[4]{82}$. Express your answer as a reduced fraction in the form $\frac{a}{b}$.

Let $a = 81$ since for $f(x) = \sqrt[4]{x}$ then $f(81) = 3$.

$$\sqrt[4]{82} \approx f(81) + f'(81)(82 - 81) = 3 + \frac{1}{4\left(\sqrt[4]{81}\right)^3}(1) = 3 + \frac{1}{4 \cdot 27} = 3 + \frac{1}{108} = \frac{325}{108}$$

By the way, this fraction has decimal representation of 3.00926 and using the calculator, we get $\sqrt[4]{82} = 3.00922$ which agrees with our approximation up to 4 decimal places. This is probably due to the fact that 81 and 82 are 'close' together.

4. A rectangle is inscribed under the graph of $y = 10 - 2x^2$. Find the dimensions of the rectangle that create the rectangle with maximum area. What is the area of that rectangle?



Let $A = 2xy = 2x(10 - 2x^2) = 20x - 4x^3$ , now let

$$A'(x) = 20 - 12x^2 = 0 \rightarrow 20 = 12x^2 \rightarrow x = \pm\sqrt{\frac{5}{3}}$$

Take x positive, so $A_{min} = 2\sqrt{\frac{5}{3}}(10 - 2\left(\sqrt{\frac{5}{3}}\right)^2) = 2\sqrt{\frac{5}{3}}\left(10 - \frac{2 \cdot 5}{3}\right) = 2\left(\sqrt{\frac{5}{3}}\right)\left(\frac{20}{3}\right) \approx$ 17.2

5. Find the number(s) in the given interval that satisfies the conclusion of the Mean Value theorem, for: $f(x) = x^3 - 6x^2 + 9x + 3$ in $[0,4]$.

$f(4) = 4^3 - 6 \cdot 4^2 + 36 + 3 = 64 - 96 + 36 + 3 = 7$ and

$$f(0) = 3$$

So $\frac{f(4)-f(0)}{4-0} = \frac{7-3}{4} = 1$

Now set $f'(x) = 3x^2 - 12x + 9 = 1 \to 3x^2 - 12x + 8 = 0$

Which does not factor so using the quadratic formula:

$x = \frac{-(-12)\pm\sqrt{(-12)^2-4(3)(8)}}{6} \to \frac{6\pm2\sqrt{3}}{3} \to x \approx 3.16$ and $0.845$ which both qualify as answers since they are both in the given interval.

Again, on an exam I would expect you could factor and not have to use the quadratic formula.